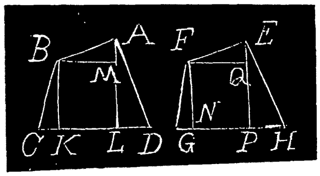
Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

Let $ABCD$, $EFGH$ be the two quadrilaterals; $BC=FG, AB=EF, AD=EH$, $\angle BCD=\angle FGH$, $\angle ADC=\angle EHG$. Draw BK, AL perpendicular to CD; FN, EP perpendicular to GH; BM perpendicular to AL; FQ perpendicular to EP.



Right triangles BCK and FGN are equal, also right triangles ALD and EPH, having hypotenuse and acute angle of one equal to hypotenuse and acute angle of other.

$\therefore$ $BK=FN$, $AL=EP$, also $AL-BK=AM=EP-FN=EQ$.

$\therefore$ right triangles ABM=right triangle FEQ; since $AB=FE$ and $AM=EQ$,

$\therefore$ $BM=FQ$. $\therefore$ $BM=KL=FQ=NP$. $\therefore$ $BKLM=FNPQ$.

$\therefore$ $BCK+ADL+ABM+BKLM=FGN+EPH+FEQ+FNPQ$.

$\therefore$ $ABCD=EFGH$.

Also solved by *J. SCHEFFER*.

170. Proposed by CHARLES C. CROSS, Whaleyville, Va.

If p, q, r are the distances of the orthocenter from the sides, prove that

$$4\left[\frac{a}{p}+\frac{b}{q}+\frac{c}{r}\right]=\left[\frac{a}{p}+\frac{b}{q}-\frac{c}{r}\right]\left[\frac{a}{p}-\frac{b}{q}+\frac{c}{r}\right]\left[-\frac{a}{p}+\frac{b}{q}+\frac{c}{r}\right].$$

Solution by MARCUS BAKER, U. S. Geological Survey, Washington, D. C.

From well known theorems we have

$$\begin{array}{lcl} a=2R\sin A & & p=2R\cos B\cos C \\ b=2R\sin B & \text{and} & q=2R\cos C\cos A \\ c=2R\sin C & & r=2R\cos A\cos B \end{array}$$

Whence $a/p=\tan B+\tan C$, $b/q=\tan C\tan A$, $c/r=\tan A+\tan B$.

Therefore, $a/p+b/q+c/r=2(\tan A+\tan B+\tan C)$.

$$-a/p+b/q+c/r=2\tan A$$

$$a/p-b/q+c/r=2\tan B$$

$$a/p+b/q-c/r=2\tan C.$$

Since $\tan A+\tan B+\tan C=\tan A\tan B\tan C$, the theorem is proved.

Also demonstrated by *L. C. WALKER*, *J. SCHEFFER*, *H. C. WHITAKER*, *G. B. M. ZERR*, and the *PROPOSER*.

171. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

Find the nearest distance of the parabola $y^2=16x$ and the ellipse $16x^2+9y^2-160-144y+832=0$.

Solution by the PROPOSER.

Let the equation of the parabola be $y^2=4px$, and that of the ellipse

$$\frac{(y-\beta)^2}{b^2}+\frac{(x-\alpha)^2}{a^2}=1.$$

A normal to the ellipse expressed by the tangent of the angle it makes with the axis of x has the equation

$$y-\beta=m(x-\alpha)-\frac{(a^2-b^2)m}{\sqrt{(b^2m^2+a^2)}},$$

and in the case of the parabola $y=mx-2pm-pm^3$. Since the shortest distance is measured off on a common normal, the two equations should be identical, and therefore

$$\beta-m\alpha-\frac{(a^2-b^2)m}{\sqrt{(b^2m^2+a^2)}}=-2pm-pm^3.$$

From this equation m is to be found. It leads to an equation of the 8th degree, which for numerical values presents no difficulty.

Substituting now the equation of the normal in both the equations of the parabola and ellipse, we find the co-ordinates of the points of intersection. Denoting these by x', y', and x'', y'', we find for the shortest distance the expression $\sqrt{[(x'-x'')^2+(y'-y'')]^2}$.

The numerical equations given having a common value, x lying between 6 and 7, and intersecting, therefore present no suitable example.

Also solved by *G. B. M. ZERR.*

CALCULUS.

129. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

Among all quadrilaterals inscribed in an ellipse, to determine that which contains the greatest area.

Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Walton furnishes a solution of this interesting problem, which for its elegance and simplicity I reproduce here.

Let the equation of the ellipse be $x^2/a^2+y^2/b^2=1$, and let the angular points of the quadrilateral be (x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4). Then, u denoting the area of the quadrilateral,

$$2u=x_2y_1-x_1y_2+x_3y_2-x_2y_3+x_4y_3-x_3y_4+x_1y_4-x_4y_1.$$

$$\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}=1,\quad \frac{x_2^2}{a^2}+\frac{y_2^2}{b^2}=1,\quad \frac{x_3^2}{a^2}+\frac{y_3^2}{b^2}=1,\quad \frac{x_4^2}{a^2}+\frac{y_4^2}{b^2}=1.$$